Phillips' Hypothesis for Eddy Viscosity

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The heart of the dynamics of turbulent shear flow is the mechanism that underlies the maintenance of Reynolds shear stress. For many years it has been customary to hypothesize a relation between the local Reynolds stress and the local mean velocity field by the use of either an eddy viscosity or a mixing length defined in one way or another. However, detailed laboratory experiments employing hot-wire anemometers in low speed air flows have demonstrated in the last two decades that any such relation was erroneous in principle. These experiments showed that the Reynolds stress is not a local property but rather one of the entire flow field. Townsend's book (*7*) is a good summary of what can be gleaned from the hot-wire anemometer studies.

The apparent shortcomings of local, simple gradient transport concepts have resulted in the suggestion of many new theories, such as the strain-rate approach of Townsend (*7*) and the wavelike interaction of turbulence with the mean flow approach of Phillips (*5*) which are very appealing on physical grounds. Interestingly enough, Phillips' reasoning originated in the wind-generated wave literature, an area of fluid mechanics far removed from turbulence. Miles (*4*) developed a theory to explain the generation of gravity waves by wind; he found that the interaction between waves and wind is restricted to that layer where the wave-phase velocity component in the wind direction exactly equaled the wind velocity. Phillips generalized this idea to a turbulent shear flow by decomposing into Fourier components all the wave numbers and frequencies of a moving turbulent field. The phase or propagation velocity of each such component is by definition the ratio of frequency to wave number. Phillips introduces an extension of Miles' mechanism by assuming that the interaction between the mean flow and the turbulent fluctuations occurs only in the matched layer where the streamwise component of the phase velocity equals the local mean flow velocity. The derivation then links the incremental Reynolds stress due to each contribution from the matched layer to the local vorticity gradient. Although the derivation is rather involved, it leads to simple relation between measurable turbulence, statistical properties, and eddy viscosity. The eddy viscosity derived by Phillips is not the customary formulation of Boussinesq, but it is easily related to the latter. Phillips gives

$$\frac{d\tau}{dy} = \nu_e \frac{d^2U}{dy^2} \qquad (1)$$

where $\tau = -\overline{uv}$. Equation (1) can be integrated by parts to give

$$\tau(y) = \nu_e(y)\frac{dU}{dy} - \int_{y_0}^{y} \frac{d\nu_e(y)}{dy}\frac{dU}{dy}\,dy \qquad (2)$$

Phillips' mechanism leads to an eddy viscosity ν_e which is a proportionality constant between the stress gradient and the mean strain-rate gradient. If ν_e is a constant in a particular turbulent shear flow, then the last term of Equation (2) vanishes and the conventional expression for eddy viscosity is obtained. Furthermore, the mechanism relates ν_e to measurable physical properties of the turbulence; thus

$$\nu_e = A\,\overline{v^2}\,\Theta \qquad (3)$$

where A is a number less than π which indicates the degree of anisotropy of the energy-containing eddies, and Θ the convected integral time scale of the lateral fluctuation velocity which has a mean square magnitude $\overline{v^2}$.

Phillips' model shows notable physical motivation and traces the time history of the turbulence with measurable turbulent properties. Inherent in his analysis is the importance of the energy-containing eddies in supporting the Reynolds shear stress. He assumes that the thickness of the critical layer where all the vorticity is conserved is small compared with the scale of variation of the mean velocity profile. Therefore, by leaving out the contributions to Reynolds stress from the outer modes of his critical layer, he deals only with large, energy-containing eddies. This indicates that Equation (3) should be applicable in free shear flows and in a region of the boundary-layer flows away from the walls where the energy-containing eddies are dominant. Even if some modes of the energy are not accounted for, the history of an eddy is traced accurately. Several experimental tests of Phillips' hypothesis and recent comments by Townsend (*8*) on the permanence and stability of energy-containing eddies support this idea favorably.

Phillips tested the analysis with experimental anemometer data obtained by Davis, Fisher, and Barratt (*3*) in the mixing region of an air jet, and he concluded that these data were consistent with the analytical prediction. For the jet flow mixing region $A = 0.17$ (*6*), but Phillips states that, depending on the shape of the turbulent eddy, the precise value may vary somewhat from one turbulent shear flow to another.

Baldwin and Haberstroh (*1*) tested Phillips' hypothesis in fully developed pipe flow of air at four pipe Reynolds numbers varying between 2.55 to 5.73×10^5. They found that A averaged 0.33 with no systematic Reynolds number trend.

TABLE 1. PHILLIPS' EDDY VISCOSITY CONSTANT

	A	Anisotropy*
Mixing zone of a jet	0.17	18:1
Fully developed pipe flow	0.33	9.5:1
Homogeneous shear flow	0.55	5.7:1
Perfect isotropy (theoretical value)	3.14	1:1

* $A = \pi$ (anisotropy)$^{-1}$ as reported by Phillips (*6*, p. 259).

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Recently Champagne, Harris, and Corrsin (2) have measured turbulence properties in a nearly homogeneous shear flow where far downstream of a shear-turbulence generator, the turbulence has reached a nearly homogeneous asymptotic condition with constant values of the mean velocity gradient and the one-point turbulence moments. The present note interprets the experimental results in terms of Phillips' hypothesis. The experimentally measured value of the eddy viscosity ν_e is found to be 0.0144 sq. ft./sec. The Eulerian space-time correlation of the axial velocity fluctuations in the apparent convective frame was fit with an exponential curve. The convective integral scale L_τ was then read as the value of τ where the peak correlations dropped to the value of $1/e$. This procedure is identical to that followed by Phillips (5) in interpreting the jet mixing region data of Davis, Fisher, and Barratt (3) and by Baldwin and Haberstroh (1) in interpreting fully developed pipe flow data. Θ is found to be 0.098 sec. The value of A for nearly homogeneous shear flow was calculated from Equation (3) to be 0.55.

Three different experimental tests of Phillips' model give values of A of the same order and these values indicate the anisotropy of the energy-containing eddies for different turbulent shear flows as shown in Table 1.

The elongations of the energy-containing eddies are represented reasonably by the experimental A values. The descending degree of anisotropy for the turbulent shear flows that are considered in Table 1 shows that Phillips' mechanism, Equation (3), is consistent with the available experimental observations.

NOTATION

A = dimensionless constant, Equation (3)
e = Naperian logarithm base, 2.718
L_τ = convective integral scale of axial turbulent velocity from space-time data, sec.
U = mean velocity, ft./sec.
$\overline{v^2}$ = mean square of lateral turbulent velocity, sq.ft./sec.^{2}
y = lateral coordinate, ft.
Θ = convective integral scale of lateral turbulent velocity from space-time data, sec.
ν_e = kinematic eddy viscosity, sq.ft./sec.
τ = turbulent shear stress, lb.$_f$/sq.ft.
τ = time delay in space-time correlation data, sec.

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On the Stretching of Dilute Polymer Solutions

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In a recent communication, Denn and Marrucci examined the concept of a limiting stretch rate in viscoelastic liquids (1). Using as a basis a convected Maxwell model (2) with constant coefficients, these authors found that the manifestation of such a limiting rate is intimately connected with the total time the pure stretching deformation is imposed. Thus, for the well-known limiting condition $2\lambda\Gamma = 1$, where λ = relaxation time, Γ = stretch rate, Denn and Marrucci showed that a time $t = 15\lambda$ was required for the stress (in an initially unstressed fluid) to increase an order of magnitude above the Newtonian value. Such considerations are of especial importance in understanding turbulent drag reduction (3), as the concept of a limiting stretch rate has often been used to explain this phenomenon (3 to 6).

The constitutive equation used in the aforementioned study, namely

$$\tau^{ij} + \lambda \frac{\overline{D}\tau^{ij}}{Dt} = 2\mu d^{ij} \tag{1}$$

$$\frac{\overline{D}\tau^{ij}}{Dt} = \frac{\partial \tau^{ij}}{\partial t} + v^k \tau^{ij}{}_{;k} - \tau^{kj} v^i{}_{;k} - \tau^{ik} v^j{}_{;k} \tag{2}$$

$$T^{ij} = -pg^{ij} + \tau^{ij} \tag{3}$$

suffers from a number of limitations. In particular, Equations (1) to (3) predict both a constant viscosity and a zero secondary normal stress difference in steady shearing flow, in disagreement with generally accepted behavior (7, 8). In light of these facts, and also the recent studies indicating the importance of the secondary normal stress difference in nonviscometric flows (9 to 11), it was felt worthwhile to repeat the Denn-Marrucci analysis, using a more realistic model.

CONSTITUTIVE EQUATION

Gordon and Schowalter recently presented a modification of the dumbbell theory of dilute polymer solutions (12), based on a structured fluid theory of Ericksen (13). Gordon and Everage used this result to obtain an explicit constitutive equation (14), namely

$$\tau^{ij} + \theta \frac{\overline{\overline{D}}\tau^{ij}}{Dt} = \frac{2Nc}{M} kT\theta(1 - \epsilon) d^{ij} \tag{4}$$

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